

The Legacy of M. P. Bronstein: on relativistic wave equations for spin 2 fields and some comments

Diego Julio Cirilo-Lombardo

*Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research 141980,
Dubna(Moscow Region), Russian Federation*

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Abstract

We briefly give a very simple picture about one of the most remarkable results of Matvej Petrovich Bronstein concerning the quantization of the gravitational waves showing also that the linearized Einstein equations of the paper: Phys.Rev. D65 (2002) 104005 are the same Bronstein's equations given 66 years before.

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I. M.P.BRONSTEIN, LINEARIZED EINSTEIN EQUATIONS AND QUANTIZATION OF GRAVITATIONAL WAVES

As it is well known, the brilliant Soviet physicist M.P. Bronstein had a wide interest in astrophysics, cosmology, semiconductors and nuclear physics but was particularly attracted to gravitation[1,5]. Without entering too many details (we refer to [1,5] and references therein) one of the most significant Bronstein's works " The quantization of the gravitational waves" was published in 1936 [2], being the first deep investigation in quantum gravity. The main goal of his paper [2] was the presentation of the Einstein equations in the following form:

$$\frac{1}{c} \frac{\partial H_{ij}}{\partial t} = -\varepsilon_{ikl} \frac{\partial E_{lj}}{\partial x_k}, \quad \frac{\partial H_{ij}}{\partial x_i} = 0 \quad (1)$$

$$\frac{1}{c} \frac{\partial E_{ij}}{\partial t} = \varepsilon_{ikl} \frac{\partial H_{lj}}{\partial x_k}, \quad \frac{\partial E_{ij}}{\partial x_i} = 0 \quad (2)$$

where E_{ij} and H_{lj} are symmetrical and traceless tensors defined as (in Bronstein's work original notation, $x_0 = it$)

$$E_{ij} = R_{4i4j} = \frac{1}{4} \varepsilon_{ikl} \varepsilon_{jmn} R_{klmn} \quad (3)$$

$$H_{ij} = \frac{i}{2} \varepsilon_{imn} R_{mn4j} = \frac{i}{2} \varepsilon_{imn} R_{4jmn} \quad (4)$$

and R_{klmn} is the curvature tensor (notice that this fact is not accidental, it follows from the general form of the relativistic wave equations for massless fields [4])

More recently in the paper [3] by E.T. Newman the following equations appear (in Newman's work notation)

$$\nabla_a W^{abcd} = 0 \quad (5)$$

where in the above equations the following selfdual quantity was defined

$$W^{abcd} = C^{abcd} - iC^{*abcd} \quad (6)$$

where C^{abcd} is the Weyl tensor, that as it is well known, it coincides with the Riemann tensor in the case of Einstein equations in vacuum without cosmological constant. Consequently eq.(5) are nothing more than, namely, the second Bianchi identities. Then, these are exactly the Bronstein equations (1,2) because after the 3+1 splitting the complex field equations (5) take the form

$$\nabla_0 W^{0i0k} + i\varepsilon^{ijl} \nabla_j W^{0l0k} = 0 \quad (7)$$

$$\nabla_i W^{i0j0} = 0 \quad (8)$$

where now if we redefine W^{0i0k} as

$$W^{i0j0} \equiv E^{ij} + iH^{ij} \quad (9)$$

(in complete analogy within the case of the Maxwell equation in flat space) we immediately obtain

$$\nabla_0 (E^{ik} + iH^{ik}) + i\varepsilon^{ijl} \nabla_j (E^{lk} + iH^{lk}) = 0 \quad (10)$$

$$\nabla_i (E^{ij} + iH^{ij}) = 0 \quad (11)$$

that are nothing more than the Bronstein equations:

$$\nabla_0 E^{ik} = \varepsilon^{ijl} \nabla_j H^{lk}, \quad \nabla_i E^{ij} = 0 \quad (12)$$

$$\nabla_0 H^{ik} = -\varepsilon^{ijl} \nabla_j E^{lk}, \quad \nabla_i H^{ij} = 0 \quad (13)$$

(considering that linealization means flat metric connection, as pointed out also in [3]). However and surprisingly into the paper [3] (see Section III: Linealized GR) no mention about the Bronstein work nor his specific reference [2] were given. Only, the author of [3] redefines again the expression (9) as $E^{ij} + iH^{ij} \equiv Z^{ij}$ rewriting equation (7) and (8) (alternatively (10) and (11)) as

$$\partial_t \overleftrightarrow{Z} + i \cdot \text{curl} \overleftrightarrow{Z} = 0 \quad (14)$$

$$\text{div} \overleftrightarrow{Z} = 0 \quad (15)$$

where $\overleftrightarrow{Z} = \overleftrightarrow{E} + i\overleftrightarrow{H}$ was defined in [3] as "dyadic form". The equations were concluded in [3] with the following claim (the numbers of the formulas correspond to our paper):

Remark 1 *It has been well known for many years that Eq.(5) can be viewed as the linear Einstein equations. We, however, have not been able to find in the literature the particular form, (14,15), that so mimics the Maxwell equations. Though we would be surprised, it might well be new."*

Sometimes certain phrases do not ever lose value despite their age, social or historical context depending only on the wisdom of the speaker:

"What was, will be again, what has been done, will be done again, and there is nothing new under the sun! Take anything which people acclaim as being new: it existed in the centuries preceding us. No memory remains of the past, and so it will be for the centuries to come – they will not be remembered by their successors." King Solomon in Ecclesiastes 1, 9-11

As always, the greatness of a person is in his capacity to recognize their failures, successes and mistakes. A possessor of this greatness (in my humble opinion) is without any doubt the referee of this paper and author of reference [3]. He himself has asked me to add a comment, and I have the pleasure to do it:

Remark 2 *I am delighted to have it pointed out that the work of the brilliant Russian physicist M. P. Bronstein preceded my work by 66 years."*

Remark 3 *The author of reference [3] kindly inform us about that in ref.[6] there is the construction of the gravitational wave equations given by M. Bronstein but no mention about ref. [2]"*

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